Large-scale magnetic fields from inflation due to a CPT-even Chern-Simons-like term with Kalb-Ramond and scalar fields

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Abstract

We investigate the generation of large-scale magnetic fields due to the breaking of the conformal invariance in the electromagnetic field through the CPT-even dimension-six Chern-Simons-like effective interaction with a fermion current by taking account of the dynamical Kalb-Ramond and scalar fields in inflationary cosmology. It is explicitly demonstrated that the magnetic fields on 1Mpc scale with the field strength of $\sim 10^{-9}$G at the present time can be induced.

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I. INTRODUCTION

Magnetic fields with the field strength 10^{-7}–10^{-6}G on 10kpc–1Mpc scale in clusters of galaxies as well as $\sim 10^{-6}$G on 1–10kpc scale in galaxies of all types and at cosmological distances are observed (for reviews on the cosmic magnetic fields, see [1, 2]). However, the origin of the cosmic magnetic fields, in particular the large-scale magnetic fields in clusters of galaxies, is not well understood yet. It is known that the dynamo amplification mechanism [3] can amplify very weak seed magnetic fields up to $\sim 10^{-6}$G in spiral galaxies, but its effectiveness in galaxies at high redshifts and clusters of galaxies is not confirmed. Other mechanisms to generate the cosmic magnetic fields exist, such as astrophysical processes [4, 5], cosmological phase transitions [6] and primordial density perturbations before or at the epoch of recombination [7]. However, it is difficult for these mechanisms to induce the magnetic fields on megaparsec scales with sufficient field strengths to explain these observed in galaxies and clusters of galaxies without the dynamo amplification mechanism.

It is considered that the most natural origin of the large-scale magnetic fields is from electromagnetic quantum fluctuations at the inflationary stage [8][1]. The reason is that inflation naturally extends the scale of the electromagnetic quantum fluctuations to the one larger than the Hubble horizon. The conformal invariance in the electromagnetic field must have been broken at the inflationary stage in order for electromagnetic quantum fluctuations to be produced during inflation [10][2]. This is because the ordinary Maxwell theory is conformally invariant, whereas the metric is conformally flat in the Friedmann-Lemaître-Robertson-Walker (FLRW) space-time. It should be cautioned that this does not apply if the FLRW background has nonzero spatial curvature such as an open FLRW background [12]. Various mechanisms of the breaking conformal invariance in the electromagnetic field have been proposed in the literature, such as those due to the non-minimal gravitational coupling [8, 13], Weyl-Maxwell fields coupling [14], coupling to a scalar field [15–24], generic coupling to a time-dependent background field [25], nonlinear electrodynamics [26], photon-graviphoton mixing [27], gravitoelectromagnetic inflationary formalism [28], conformal anomaly induced by quantum effects [29], spontaneous breaking of the Lorentz invariance [30], Lorentz violat-

[1] The back reaction of the generated magnetic fields on inflation has been argued [9].

[2] The effect of the breaking of the conformal flatness due to scalar metric perturbations at the end of inflation has also been discussed [11].

ing term [31], Lorentz gauge-breaking term [32], noncommutative field theory [33], preferred minimal length [34], cosmic defect [35], bouncing cosmology [36], and Hořava-Lifshitz gravity [37]. For other breaking mechanisms and references, see a recent review in Ref. [2]. Moreover, the complementary studies of magnetic catalysis in the gauge Higgs-Yukawa model and neutrino propagation in a strongly magnetized medium, also in view to their cosmological impact, have also been studied in Refs. [38–40]. In addition, it is interesting to mention that a lower bound on the magnetic field strength in the hot universe has been recently obtained in Ref. [41].

Recently, the CPT-even dimension-six Chern-Simons-like effective interaction between a fermion current and the electromagnetic field in inflationary cosmology has been studied to induce the cosmological birefringence [42, 43], baryon number asymmetry [44], and large-scale magnetic field [45], respectively. In a related work [46], the generation of large-scale magnetic fields during inflation was examined in a Lorentz violating theory of Electrodynamics due to a Chern-Simons term coupling the $U(1)$ gauge field to an external four-vector, proposed in Ref. [47]. Furthermore, the CPT-even dimension-six Chern-Simons-like term with including the dynamical Kalb-Ramond and scalar fields was investigated to produce the cosmological birefringence [48]. Spectral dependence of the cosmic microwave background (CMB) polarization and parity has also been discussed in Ref. [49]. The estimation of relic magnetic fields from CMB temperature correlations has been executed in Ref. [50]. Moreover, cosmological consequences of the existence of a Kalb-Ramond field have been studied in Ref. [51]. In addition, the role of spin and polarization in gravity have been considered in Ref. [52] and limits on cosmological birefringence from the UV polarization of distant radio galaxies have been examined in Ref. [53]. To search other cosmological ingredients from this term, in this paper we explore the generation of large-scale magnetic fields due to the breaking of the conformal invariance in the electromagnetic field through the CPT-even dimension-six Chern-Simons-like effective interaction with a fermion current by taking account of the dynamical Kalb-Ramond and scalar fields in inflationary cosmology.

The paper is organized as follows. In Sec. II, we describe our model and derive equations of motion for the $U(1)$ gauge field. In Sec. III, we consider the evolution of the $U(1)$ gauge field and estimate the present strength of the large-scale magnetic fields. Finally, conclusions are given in Sec. IV.

II. THE MODEL

Our model action is given by [48]

$$S = \int d^4x\sqrt{g}\Big[-\frac{1}{2}\epsilon\phi^2 R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)$$
$$-\frac{\xi_1}{6\phi^2}\tilde{H}_{\mu\nu\alpha}\tilde{H}^{\mu\nu\alpha} + \frac{\xi_2}{\phi^2}j_\mu\left(A_\nu\tilde{F}^{\mu\nu} + \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}\partial_\nu B_{\alpha\beta}\right) - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}\Big], \tag{2.1}$$

where g is the determinant of the metric tensor $g_{\mu\nu}$, $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ is the electromagnetic field strength tensor, $\tilde{F}^{\mu\nu} = (1/2)\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$ is the dual of $F_{\mu\nu}$ with $\epsilon^{\mu\nu\alpha\beta} = (1/\sqrt{g})e^{\mu\nu\alpha\beta}$ being the Levi-Civita tensor normalized by $e^{0123} = +1$, R is the Ricci scalar, ϕ is a dynamical scalar field, and $B_{\mu\nu}$ is the Kalb-Ramond fields with the modified field strength $\tilde{H}_{\mu\nu\alpha} = \partial_{[\mu}B_{\nu\alpha]} + A_{[\mu}F_{\nu\alpha]}$. We use units of $k_{\rm B} = c = \hbar = 1$ and adopt Heaviside-Lorentz units of electromagnetism.

The following set of equations of motion can be obtained by varying the action with respect to ϕ, $g_{\mu\nu}$, $B_{\mu\nu}$ and A_μ:

$$\epsilon\phi R = D_\mu\partial^\mu\phi - \frac{\partial V}{\partial\phi} + \frac{\xi_1}{3\phi^3}\tilde{H}^2 - 2\frac{\xi_2}{\phi^3}j_\mu\left(A_\nu\tilde{F}^{\mu\nu} + \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}\partial_\nu B_{\alpha\beta}\right), \tag{2.2}$$

$$\epsilon\phi^2 G_{\mu\nu} = \left[\frac{1}{2}\left(\partial_\alpha\phi\right)^2 + V(\phi)\right]g_{\mu\nu} - \partial_\mu\phi\partial_\nu\phi + \frac{\xi_1}{6\phi^2}\tilde{H}^2 g_{\mu\nu} + \left(\frac{1}{4}F^2 g_{\mu\nu} - F_{\mu\alpha}F^\alpha_\nu\right)$$
$$+\epsilon(D_\nu D_\mu\phi^2 - D^\sigma D_\sigma\phi^2 g_{\mu\nu}) - \frac{1}{\phi^2}\tilde{H}_{\mu\alpha\beta}\tilde{H}^{\alpha\beta}_\nu, \tag{2.3}$$

$$D_\mu\left(\frac{\xi_1}{\phi^2}\tilde{H}^{\mu\nu\alpha} + \frac{\xi_2}{2\phi^2}\epsilon^{\mu\nu\alpha\beta}j_\beta\right) = 0, \tag{2.4}$$

$$D_\nu F^{\nu\mu} - D_\nu\left(\frac{2\xi_1}{\phi^2}\tilde{H}^{\nu\alpha\mu}A_\alpha + \frac{\xi_2}{\phi^2}\epsilon^{\beta\alpha\nu\mu}j_\beta A_\alpha\right) = \frac{\xi_1}{\phi^2}\tilde{H}^{\mu\nu\alpha}F_{\nu\alpha} - \frac{\xi_2}{\phi^2}j_\nu\tilde{F}^{\nu\mu}. \tag{2.5}$$

Since $\tilde{H}^{\mu\nu\alpha}$ is a totally antisymmetric tensor, we can write $\tilde{H}^{\mu\nu\alpha} = \epsilon^{\mu\nu\alpha\beta}T_\beta$, where T_β is a vector with mass dimension three. Thus, Eq. (2.4) is rewritten to

$$\epsilon^{\mu\nu\alpha\beta}\partial_\mu\left(\frac{\xi_1}{\phi^2}T_\beta + \frac{\xi_2}{2\phi^2}j_\beta\right) = 0. \tag{2.6}$$

Focusing on the space-time manifold with the first trivial homology group, any closed one-form is an exact one-form. Therefore, from Eq. (2.6), we can express the torsion field as

$$\frac{1}{\phi^2}\left(\xi_1 T_\beta + \frac{\xi_2}{2}j_\beta\right) = \partial_\beta\Phi, \tag{2.7}$$

where Φ is a dimensionless pseudo-scalar. With the help of Eq. (2.7), we can further simplify the equations of motion (2.2), (2.3) and (2.5) to be

$$\epsilon\phi R = D_\mu \partial^\mu \phi - \frac{\partial V}{\partial \phi} - \frac{2\phi}{3\xi_1} \left(\partial_\mu \Phi\right)^2 + \frac{\xi_2^2}{2\xi_1 \phi^3} \left(j_\mu\right)^2 \,, \tag{2.8}$$

$$\begin{aligned}\epsilon\phi^2 G_{\mu\nu} =& \left[\frac{1}{2}\left(\partial_\alpha \phi\right)^2 + V(\phi)\right] g_{\mu\nu} - \partial_\mu \phi \partial_\nu \phi + \epsilon (D_\nu D_\mu \phi^2 - D^\sigma D_\sigma \phi^2 g_{\mu\nu}) \\ &+ \frac{1}{\xi_1 \phi^2}\left[\phi^4 \left(\partial_\alpha \Phi\right)^2 - \xi_2 \phi^2 j_\alpha \partial^\alpha \Phi + \frac{\xi_2^2}{4}\left(j_\mu\right)^2\right] g_{\mu\nu} \\ &+ \left(\frac{1}{4} F^2 g_{\mu\nu} - F_{\mu\alpha} F^\alpha_\nu\right) - 2\frac{\xi_1}{\phi^2}\left(\frac{\phi^2}{\xi_1}\partial_\mu \Phi - \frac{\xi_2}{2\xi_1} j_\mu\right)\left(\frac{\phi^2}{\xi_1}\partial_\nu \Phi - \frac{\xi_2}{2\xi_1} j_\nu\right) \,, \end{aligned} \tag{2.9}$$

$$D_\mu F^{\mu\nu} = -4\left(\partial_\mu \Phi\right) \tilde{F}^{\mu\nu} \,, \tag{2.10}$$

respectively.

Now, we consider the simplest ϕ^4 potential, given by

$$V(\phi) = \lambda \left(\phi^2 - \phi_0^2\right)^2 + V_0 \,, \tag{2.11}$$

where V_0 and λ are both larger than zero. We take the flat FLRW space-time with the metric,

$$ds^2 = -dt^2 + a^2(t) d\boldsymbol{x}^2 \,, \tag{2.12}$$

where $a(t)$ is the scale factor. In the FLRW universe, it is reasonable to assume a homogeneous and isotropic fermion current and Kalb-Ramond field [48], i.e., $j_\mu = (j_0(t), \mathbf{0})$ and $T_\mu = (T_0(t), \mathbf{0})$. From Eqs. (2.8) and (2.9), we have

$$\partial_0 \Phi = -2\frac{\xi_1 V_0}{\xi_2 j_0} + \frac{\xi_2}{2\phi_0^2} j_0 \,. \tag{2.13}$$

For the Coulomb gauge of $A_0(t, \boldsymbol{x}) = 0$ and $\partial_j A^j(t, \boldsymbol{x}) = 0$, Eq. (2.10) becomes

$$\ddot{A}_j(t, \boldsymbol{x}) + H\dot{A}_j(t, \boldsymbol{x}) - \frac{1}{a^2}\partial_i \partial_i A_j(t, \boldsymbol{x}) - 4\frac{\dot{\Phi}}{a} e_{jik} \partial_i A_k(t, \boldsymbol{x}) = 0 \,, \tag{2.14}$$

where a dot denotes a time derivative, $H = \dot{a}/a$ is the Hubble parameter, and e_{ijk} is the totally antisymmetric tensor ($e_{123} = +1$).

III. LARGE-SCALE MAGNETIC FIELDS

A. Evolution of the $U(1)$ gauge field

We consider the case in which a slow-roll exponential inflation occurs with $a(t) = a_1 \exp\left[H_{\rm inf}(t - t_1)\right]$, where a_1 is the scale factor at the time t_1 when a comoving wavelength

$2\pi/k$ of the $U(1)$ gauge field first crosses outside the horizon during inflation, $k/(a_1 H_{\rm inf}) = 1$, and $H_{\rm inf}$ is the Hubble constant at the inflationary stage.

It follows from the quantization of the $U(1)$ gauge field $A_\mu(t, \boldsymbol{x})$ that $A_i(t, \boldsymbol{x})$ is expressed as

$$A_i(t, \boldsymbol{x}) = \int \frac{d^3k}{(2\pi)^{3/2}} \left[\hat{b}(\boldsymbol{k}) A_i(t, \boldsymbol{k}) e^{i\boldsymbol{k}\cdot\boldsymbol{x}} + \hat{b}^\dagger(\boldsymbol{k}) A_i^*(t, \boldsymbol{k}) e^{-i\boldsymbol{k}\cdot\boldsymbol{x}} \right], \tag{3.1}$$

where $\boldsymbol{k}$ is the comoving wave number, k denotes its amplitude $|\boldsymbol{k}|$, and $\hat{b}(\boldsymbol{k})$ and $\hat{b}^\dagger(\boldsymbol{k})$ are the annihilation and creation operators which satisfy $\left[\hat{b}(\boldsymbol{k}), \hat{b}^\dagger(\boldsymbol{k}')\right] = \delta^3(\boldsymbol{k} - \boldsymbol{k}')$ and others $= 0$. In what follows, we choose the x^3 axis to lie along the spatial momentum direction $\boldsymbol{k}$ and denote the transverse directions x^I with $I = 1, 2$. We use circular polarizations expressed by the combination of linear polarizations as $A_\pm(k,t) \equiv A_1(k,t) \pm i A_2(k,t)$. From Eq. (2.14), we obtain

$$\ddot{A}_\pm(k,t) + H\dot{A}_\pm(k,t) + \frac{k^2}{a^2} A_\pm(k,t) \mp 4\dot{\Phi}\frac{k}{a} A_\pm(k,t) = 0\,, \tag{3.2}$$

with

$$\dot{\Phi} = -\frac{\xi_1}{\xi_2}\frac{2V_0}{j_0} + \frac{\xi_2}{2\phi_0^2} j_0\,, \tag{3.3}$$

where j_0 is the fermion number density. Here, we take $j_0 = \bar{n} a^{-m}$ $(m > 0)$, $\phi^2 = M_{\rm Pl}^2 = 1/(8\pi G)$, $H_{\rm inf} = 10^{10}$GeV, and the comoving scale $L = 2\pi/k = 1$Mpc. Since there exists no analytic solution of Eq. (3.2), we investigate the numerical solutions for $m = 1$, 2 and 3. Note that $m = 3$ corresponds to the conventional property.

By using $k/a = (k/a_1)e^{-H_{\rm inf}(t-t_1)} = H_{\rm inf} e^{-H_{\rm inf}(t-t_1)}$, the equation of motion (3.2) is rewritten to

$$\begin{aligned} &\ddot{A}_\pm(k,t) + H_{\rm inf}\dot{A}_\pm(k,t) \\ &+ \left[H_{\rm inf}^2 e^{-2H_{\rm inf}(t-t_1)} \mp 4H_{\rm inf} e^{-H_{\rm inf}(t-t_1)} \left(-\frac{\xi_1}{\xi_2}\frac{2V_0}{\bar{n}} a^m + \frac{\xi_2}{2M_{\rm Pl}^2}\bar{n} a^{-m} \right) \right] A_\pm(k,t) = 0\,, \end{aligned} \tag{3.4}$$

which can be simplified to be

$$A''_\pm(k,\tilde{t}\,) + A'_\pm(k,\tilde{t}\,) + e^{-2(\tilde{t}-\tilde{t}_1)} \left[1 \mp J_1 e^{(m+1)(\tilde{t}-\tilde{t}_1)} \mp J_2 e^{(1-m)(\tilde{t}-\tilde{t}_1)} \right] A_\pm(k,\tilde{t}\,) = 0\,, \tag{3.5}$$

where the prime denotes a derivative with respect to $\tilde{t}$ and

$$\tilde{t} \equiv H_{\rm inf} t\,, \tag{3.6}$$

$$J_1 \equiv \frac{4}{H_{\rm inf}} \left[-\frac{\xi_1}{\xi_2}\frac{2V_0}{\bar{n}} \left(\frac{k}{H_{\rm inf}} \right)^m \right], \tag{3.7}$$

$$J_2 \equiv \frac{4}{H_{\rm inf}} \left[\frac{\xi_2}{2M_{\rm Pl}^2}\bar{n} \left(\frac{k}{H_{\rm inf}} \right)^{-m} \right]. \tag{3.8}$$

We take the following initial conditions at $\tilde{t}_1 = H_{\rm inf} t_1 = 1$:

$$A_\pm(k, \tilde{t}_1 = 1) = \frac{1}{\sqrt{2k}}\,, \quad A'_\pm(k, \tilde{t}_1 = 1) = \frac{H_{\rm inf}}{\sqrt{2k}}\,, \tag{3.9}$$

in order that the vacuum should reduce to the one in Minkowski space-time in the short-wavelength limit. For convenience in numerical calculations, we introduce the variable $C_\pm(k, \tilde{t}\,)$ to separate the coefficient $1/\sqrt{2k}$ from the amplitudes $A_\pm(k, \tilde{t}\,)$ as $A_\pm(k, \tilde{t}\,) = C_\pm(k, \tilde{t}\,) A_\pm(k, \tilde{t}_1) = \left(1/\sqrt{2k}\right) C_\pm(k, \tilde{t}\,)$ and $A'_\pm(k, \tilde{t}\,) = C'_\pm(k, \tilde{t}\,) A_\pm(k, \tilde{t}_1) = \left(1/\sqrt{2k}\right) C'_\pm(k, \tilde{t}\,)$. From Eq. (3.5), we find

$$C''_\pm(k, \tilde{t}\,) + C'_\pm(k, \tilde{t}\,) + e^{-2(\tilde{t}-\tilde{t}_1)} \left[1 \mp J_1 e^{(m+1)(\tilde{t}-\tilde{t}_1)} \mp J_2 e^{(1-m)(\tilde{t}-\tilde{t}_1)}\right] C_\pm(k, \tilde{t}\,) = 0\,, \tag{3.10}$$

with the initial conditions at $\tilde{t} = 1$ as $C_\pm(k, \tilde{t} = 1) = 1$ and $C'_\pm(k, \tilde{t} = 1) = H_{\rm inf}$.

In Figs. 1, 2 and 3, we depict $C_+(k, \tilde{t}\,)$ (left) and $C_-(k, \tilde{t}\,)$ (right) as functions of $\tilde{t} \equiv H_{\rm inf} t$ with a comoving scale $L = 2\pi/k = 1{\rm Mpc}$ for $n = \bar{n} a^{-1}$ ($\bar{n} = 10^{-104.36}$), $n = \bar{n} a^{-2}$ ($\bar{n} = 10^{-45.3}$), and $n = \bar{n} a^{-3}$ ($\bar{n} = 10^{-92.45}$), respectively, where $H_{\rm inf} = 10^{10}{\rm GeV}$, $V_0 = 10^{-47}$ ${\rm GeV}^4$ and $\xi_1 = \xi_2 = 1$. It is known that $H_{\rm inf} < 6.0 \times 10^{14}{\rm GeV}$ from tensor perturbations [54] with the observational data on the anisotropy of the CMB radiation [55].

We note that the evolutions of $C_+(k, \tilde{t}\,)$ and $C_-(k, \tilde{t}\,)$ depend on $\bar{n}$. To generate the magnetic fields with enough strength, we have to choose a specific value of $\bar{n}$. We also remark that the behaviors of $C_+(k, \tilde{t}\,)$ and $C_-(k, \tilde{t}\,)$ for $m = 1$ are different from those for $m > 1$. In $m = 1$, $C_+(k, \tilde{t}\,)$ approaches a constant at a large $\tilde{t}$, but $C_-(k, \tilde{t}\,)$ increases with $\tilde{t}$. On the other hand, for $m = 2$ and 3, both $C_+(k, \tilde{t}\,)$ and $C_-(k, \tilde{t}\,)$ become constants at large $\tilde{t}$. We will discuss the asymptotic behavior of $C_+(k, \tilde{t}\,)$ and $C_-(k, \tilde{t}\,)$ in Appendix A.

B. Strength of the large-scale magnetic fields

We estimate the present strength of the large-scale magnetic fields by using the numerical results for $C_\pm(k, \tilde{t}\,)$. The proper magnetic fields are given by [15]

$$B_i^{\rm proper}(t, x) = a^{-1} B_i(t, x) = a^{-2} \epsilon_{ijk} \partial_j A_k(t, x)\,, \tag{3.11}$$

where $B_i(t, \boldsymbol{x})$ are the comoving magnetic fields. The energy density in Fourier space is given by

$$\rho_B(k, t) = \frac{1}{2} \left[|B_+^{\rm proper}(k, t)|^2 + |B_-^{\rm proper}(k, t)|^2\right]\,, \tag{3.12}$$

$$|B_\pm^{\rm proper}(k, t)|^2 = \frac{1}{a^2} \left(\frac{k}{a}\right)^2 |A_\pm(k, t)|^2\,, \tag{3.13}$$

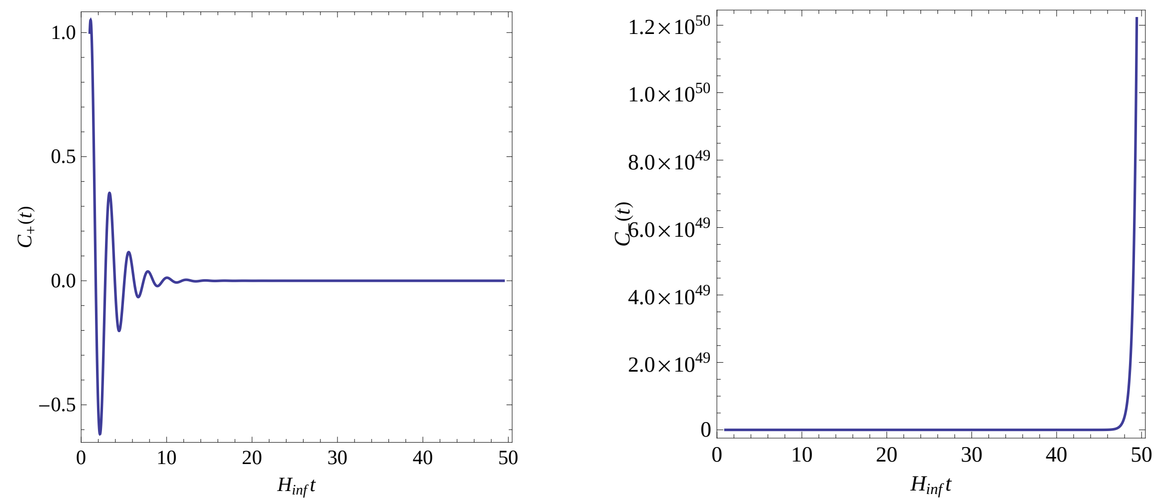

FIG. 1: $C_+(k,\tilde{t}\,)$ (left) and $C_-(k,\tilde{t}\,)$ (right) as functions of $\tilde{t} \equiv H_{\rm inf}t$ with a comoving scale $L = 2\pi/k = 1$Mpc for $n = \bar{n}a^{-1}$, where $\bar{n} = 10^{-104.36}$, $H_{\rm inf} = 10^{10}$GeV, $V_0 = 10^{-47}$ GeV^4 and $\xi_1 = \xi_2 = 1$.

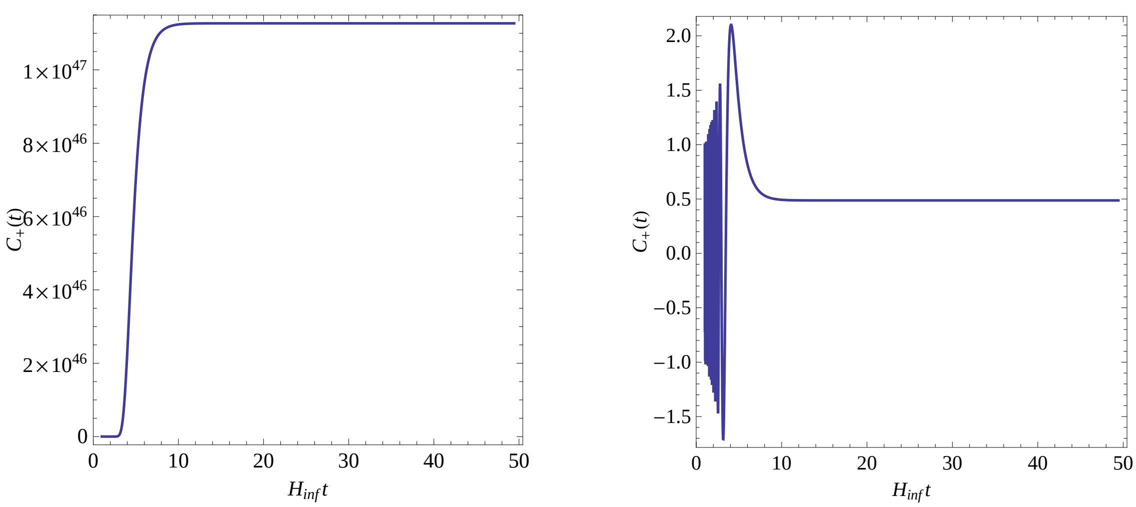

FIG. 2: Legend is the same as Fig. 1 but for $n = \bar{n}a^{-2}$ with $\bar{n} = 10^{-45.3}$.

where $B_{\pm}^{\rm proper}(k,t) \equiv B_1^{\rm proper}(k,t) \pm iB_2^{\rm proper}(k,t)$. Multiplying $\rho_B(k,t)$ by the phase space density of $4\pi k^3/(2\pi)^3$, we obtain the energy density of the proper magnetic field as

$$\rho_B(L,t) = \frac{1}{8\pi^2}\left(\frac{k}{a}\right)^4 \mathcal{I}(k,t)\,, \tag{3.14}$$

with

$$\mathcal{I}(k,t) = |C_+(k,t)|^2 + |C_-(k,t)|^2\,, \tag{3.15}$$

where $\mathcal{I}(k,t)$ can be interpreted as the amplification factor at the inflationary stage. Here, we concentrate on the situation in which after inflation the universe is reheated immediately

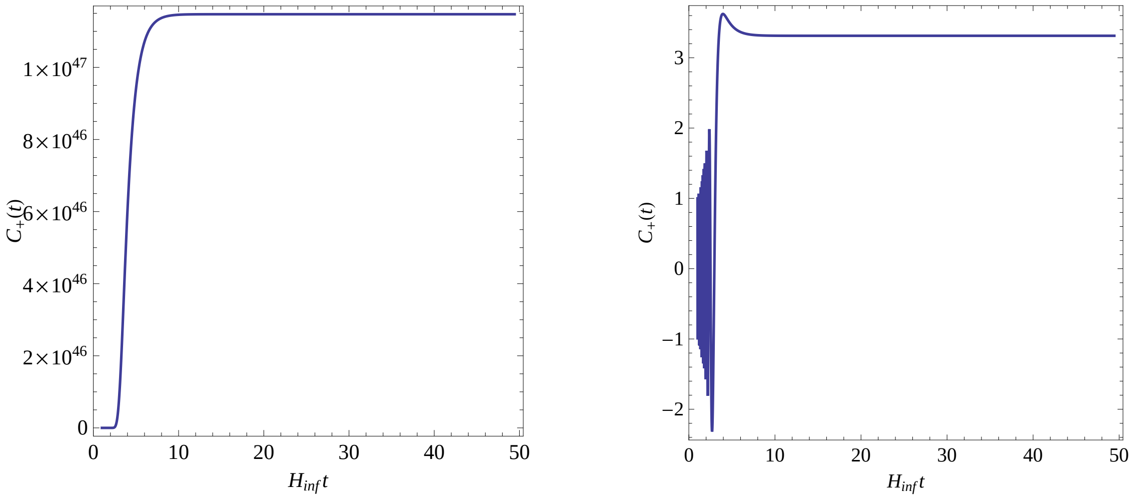

FIG. 3: Legend is the same as Fig. 1 but for $n = \bar{n}a^{-3}$ with $\bar{n} = 10^{-92.45}$.

at $t = t_{\rm R}$. The conductivity of the universe $\sigma_{\rm c}$ is negligibly small during inflation because there are few charged particles at that time. After reheating, charged particles are produced so that the conductivity immediately jumps to a large value: $\sigma_{\rm c} \gg H$. For a large enough $\sigma_{\rm c}$, magnetic fields evolve in proportion to $a^{-2}(t)$ [15]. From $B(L,t_0) = \sqrt{2\rho_B(k,t_0)}$ and Eq. (3.14), we find that $B(L,t_0) = [1/(2\pi)]\,(k/a_{\rm R})^2\,(a_{\rm R}/a_0)^2\,\sqrt{\mathcal{I}(k,t_{\rm R})}$. As a result, the present strength of the magnetic fields is described as

$$B(L,t_0) = \left(\frac{10^{20}}{1.95}\right)\frac{1}{2\pi}\left(\frac{k}{a_1}\right)^2 e^{-2N}\left(\frac{a_{\rm R}}{a_0}\right)^2\sqrt{\mathcal{I}(k,t_{\rm R})}\;[{\rm G}]\,, \tag{3.16}$$

where $a_{\rm R}/a_0 = (g_R/3.91)^{-1/3}T_{\gamma 0}/T_{\rm R}$ with $T_{\rm R}$ being the reheating temperature and $T_{\gamma 0}\,(= 2.73[{\rm K}])$ the present temperature of the CMB radiation [56], $a_{\rm R}$ and $a_0(=1)$ are the values of a at $t = t_{\rm R}$ and the present time t_0, and N is the number of e-folds between the time t_1 and $t_{\rm R}$, given by $N = 45 + \ln(L/[{\rm Mpc}]) + \ln\Xi$, where $\Xi = [30/(\pi^2 g_{\rm R})]^{1/12}\rho_{\rm R}^{1/4}/(10^{38/3}[{\rm GeV}])$, $g_{\rm R} \sim 100$ is the total number degree of freedom for relativistic particles at the reheating epoch, and $\rho_{\rm R} = (\pi^2/30)g_{\rm R}T_{\rm R}^4$ is the energy density of radiation at the reheating stage.

Using Eq. (3.16) and $H_{\rm inf}^2 = (8\pi/3)\,\rho_{\rm R}/M_{\rm Pl}^2$, we find that when $H_{\rm inf} = 10^{10}$GeV, $V_0 = 10^{-47}$ GeV^4 and $\xi_1 = \xi_2 = 1$, the generated magnetic field on 1Mpc scale at the present time is $B_0(L = 1{\rm Mpc}, t_0) = 4.1\times10^{-9}$G, 1.7×10^{-9}G and 1.7×10^{-9}G for the cases in Figs. 1, 2 and 3, respectively.

Finally, we mention constraints on the primordial magnetic fields from the Big Bang Nucleosynthesis (BBN) and CMB anisotropy measurements on small and large[3] scales, respec-

[3] There also exist constraints on the magnetic field strength on large scales from the matter density fluctu-

tively[4]. The limit on the present strength of the magnetic fields around the BBN horizon size $\sim 9.8 \times 10^{-5} h^{-1}$Mpc with $h = 0.7$ [61] is less than 10^{-6}G [62]. For the cases in Figs. 2 and 3, the present strength on the BBN horizon scale is 1.5×10^{-48}G and 1.5×10^{-48}G, respectively, which are consistent with the constraints from BBN, whereas for that in Fig. 1, it is diverging large. On the other hand, the result of $\sim 10^{-9}$G on 1Mpc scale for all cases in Figs. 1–3 is consistent with the observational upper bounds ($\sim 2 - 6 \times 10^{-9}$G) from CMB [63][5]. We also remark that future CMB polarization experiments such as PLANCK [65, 66], QUIET [67, 68], B-Pol [69] and LiteBIRD [70] can test the large-scale magnetic fields with the current amplitude $\sim 4 \times 10^{-11} - 10^{-10}$G [71].

IV. CONCLUSIONS

We have studied the generation of the large-scale magnetic fields from inflation due to the CPT-even dimension-six Chern-Simons-like effective interaction in the presence of the dynamical Kalb-Ramond and scalar fields. It has explicitly been shown that the magnetic fields on 1Mpc scale with the present amplitude of $\sim 10^{-9}$G can be generated when the number density of the fermion interacting with the electromagnetic field evolves in proportion to $a^{-m}(t)$ with $m = 1$, 2 and 3 during inflation. If the large-scale magnetic fields $\sim 10^{-9}$G are generated from inflation, the magnetic fields observed in galaxies and clusters of galaxies can be explained through only adiabatic compression without any dynamo amplification mechanism [8].

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ation parameter σ_8 [57], the fifth science (S5) run of laser interferometer gravitational-wave observatory (LIGO) [58], and Chandra X-ray galaxy cluster survey as well as Sunyaev-Zel'divich (S-Z) survey [59], which are consistent with or weaker than those from CMB.

[4] Generic property of the spectrum of large-scale magnetic fields from inflation has been discussed [60].

[5] The limit from CMB on the current strength on scales larger than the present horizon is less than 4.8×10^{-9}G [64]. To satisfy this limit, for example, one may take $n = \bar{n} a^{-2}$ with $\bar{n} = 10^{-52.44}$, $H_{\mathrm{inf}} = 10^{10}$GeV, $V_0 = 10^{-47}$ GeV^4 and $\xi_1 = \xi_2 = 1$. In this case, the present strength of the magnetic fields is 1.4×10^{-9}G on the horizon scale, while 2.9×10^{-56}G on 1Mpc.

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Appendix A: Asymptotic behavior of $C_+(k,\tilde{t}\,)$ and $C_-(k,\tilde{t}\,)$

We start from Eq. (3.10).

$$C''_\pm(k,\tilde{t}\,) + C'_\pm(k,\tilde{t}\,) + e^{-2(\tilde{t}-\tilde{t}_1)}\left[1 \mp J_1 e^{(m+1)(\tilde{t}-\tilde{t}_1)} \mp J_2 e^{(1-m)(\tilde{t}-\tilde{t}_1)}\right] C_\pm(k,\tilde{t}\,) = 0\,. \quad \text{(A1)}$$

If we take $C_\pm = e^{f_\pm}$, $g_\pm = f'_\pm$, $J = -J_1 > 0$ and $m_- = m - 1$, the field equations can be written as

$$g'(k,\tau) + g(k,\tau)^2 + g(k,\tau) \pm J e^{m_- \tau} = 0 \quad \text{(A2)}$$

when $\tau \equiv \tilde{t} - \tilde{t}_1 \to \infty$. It is easy to see that the J term dominates at large τ for all positive m_-. The homogeneous solution g_1 to above equation is, ignoring the initial time when writing τ as $\tau - \tau_0$,

$$g_1 = \frac{de^{-\tau}}{1 - de^{-\tau}} \to de^{-\tau} \quad \text{(A3)}$$

at the large time limit, with d a parameter to be fitted with the initial conditions. Hence the homogeneous part does not affect the asymptotic behavior in any significant way even the non-linear term g^2 is present. Therefore, the large time physics is controlled mainly by the algebraic equation

$$g(k,\tau)^2 + g(k,\tau) \pm J e^{m_- \tau} = 0. \quad \text{(A4)}$$

For $m = 1$ Eq. (A4) gives, the first $\pm$ sign indicating two different roots, the second $\mp$ sign indicating solutions to the $C_\pm$ equation,

$$g_\pm(k,\tau) = \frac{-1 \pm \sqrt{1 \mp 4J}}{2}. \quad \text{(A5)}$$

Therefore, we have the solution

$$C_+ \to \exp[\frac{(-1 \pm \sqrt{1-4J})\tau}{2}] \to 0 \tag{A6a}$$

for all J, with $4J > 0$ indicating the oscillating solutions. In addition,

$$C_- \to \exp[(\frac{-1+\sqrt{1+4J}}{2})\tau] \to \infty, \tag{A6b}$$

$$C_- \to \exp[(\frac{-1-\sqrt{1+4J}}{2})\tau] \to 0. \tag{A6c}$$

For $m > 1$, one has

$$C_+ \to N \exp[\pm i(2\sqrt{J}/m_-) \exp(m_-\tau/2)] \to C_0, \quad \text{a constant}, \tag{A7a}$$

$$C_- \to N \exp[\pm(2\sqrt{J}/m_-) \exp(m_-\tau/2)]. \tag{A7b}$$

It is clear that C_+ is an oscillatory solution and C_- has two solutions, one approaching to infinity (+) and the other to zero (-). A small $-1/2$ has been ignored when solving for g in above solutions. This has to do with the fact that we have ignored the homogeneous part of g equation.

In summary, we have found there is only one solution of C_+ at the large time limit and there are, however, two sets of C_- solutions for both $m = 1$ and $m > 1$. The numerical study shows that the behavior of C_- is very sensitive to the choice of the initial condition as shown in Figs. (4), (5) and (6).

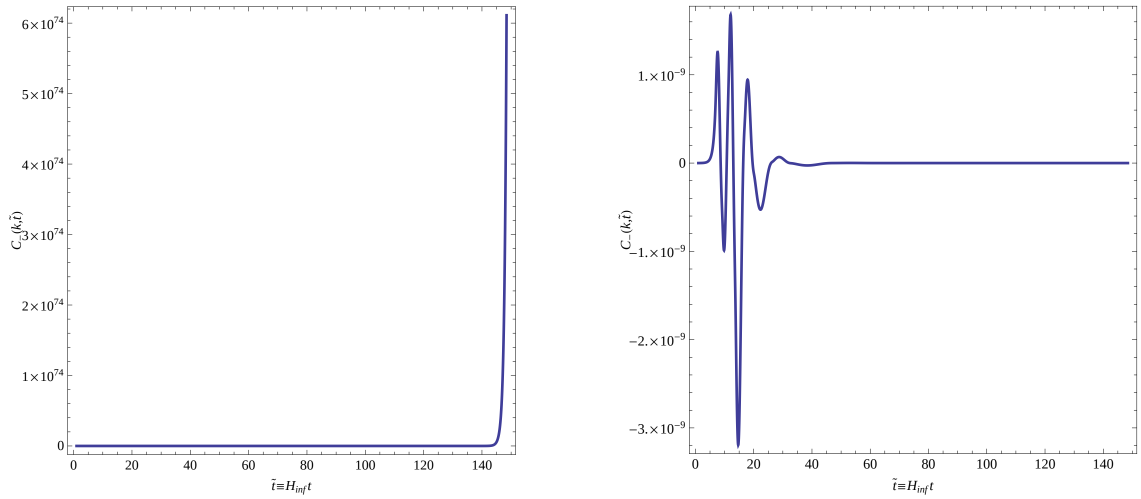

FIG. 4: $C_-(k,\tilde{t}\,)$ as functions of $\tilde{t} \equiv H_{\rm inf}t$ with a comoving scale $L = 2\pi/k = 1$Mpc for $n = \bar{n}a^{-1}$ (m=1), where $\bar{n} = 10^{-104}$, $H_{\rm inf} = 10^{10}$GeV, $V_0 = 10^{-47}$ GeV^4 and $\xi_1 = \xi_2 = 1$. The initial conditions are $C_-(\tilde{t_1} = 1) = 0$ and $C'_-(\tilde{t_1} = 1) = 10^{-12}$ (left) and $C_-(\tilde{t_1} = 1) = 0$ and $C'_-(\tilde{t_1} = 1) = 10^{-12.1}$ (right).

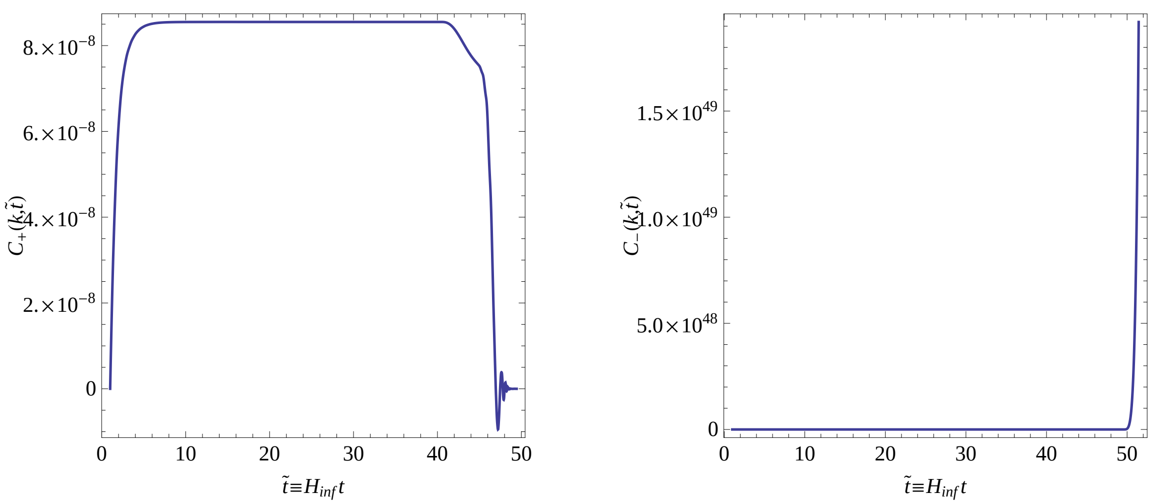

FIG. 5: $C_+(k,\tilde{t}\,)$ (left) and $C_-(k,\tilde{t}\,)$ (right) as functions of $\tilde{t} \equiv H_{\rm inf}t$ with a comoving scale $L = 2\pi/k = 1$Mpc for $n = \bar{n}a^{-4}$ (m=4), where $\bar{n} = 10^{-187}$, $H_{\rm inf} = 10^{10}$GeV, $V_0 = 10^{-47}$ GeV^4 and $\xi_1 = \xi_2 = 1$. The initial conditions are $C_+(\tilde{t_1} = 1) = 0$ and $C'_-(\tilde{t_1} = 1) = 10^{-7}$ and $C_-(\tilde{t_1} = 1) = 0$ and $C'_-(\tilde{t_1} = 1) = 10^{-7}$ (right).

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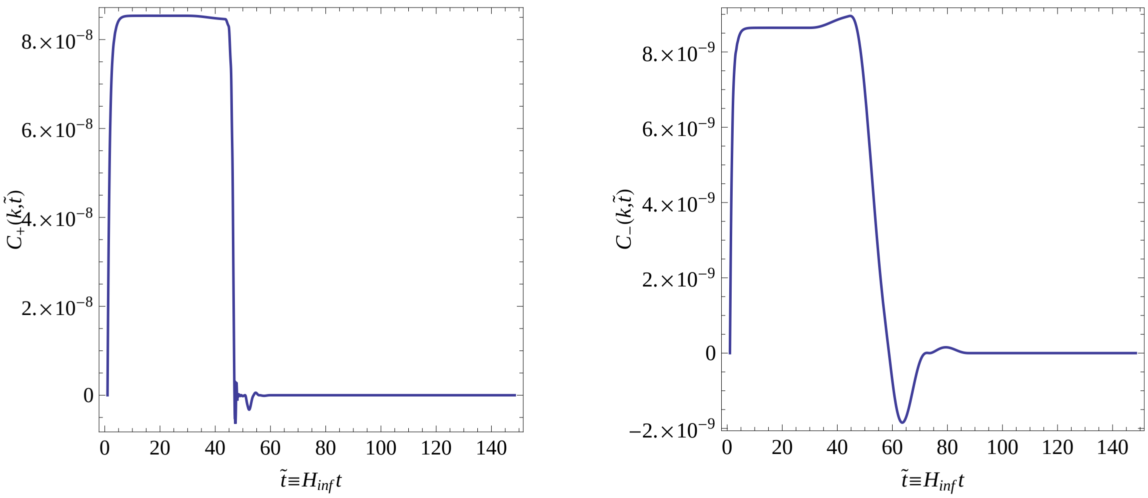

FIG. 6: Legend is the same as Fig. (5) but with initial conditions $C_+(\tilde{t_1}=1)=0$ and $C'_-(\tilde{t_1}=1)=10^{-7}$ and $C_-(\tilde{t_1}=1)=0$ and $C'_-(\tilde{t_1}=1)=10^{-8}$ (right).

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